(54) PREDICTION OF THE ULTIMATE FLEXURAL STRENGTH OF EXTERNALLY PRESTRESSED BEAMS

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1. INTRODUCTION

One of the latest developments in prestressed concrete technology has been the use of external prestressing, which may be defined as a method of prestressing where major portion of the tendons is placed outside the concrete section. The ultimate flexural analysis of such beams offers an additional difficulty, in comparison to the beams with bonded tendons. The stress increase in the external tendons beyond the effective prestress due to applied loading is member dependent rather than section dependent. It has been shown by Matupayont [1] that the eccentricity variations could have a marked influence in the ultimate strength of externally prestressed beams. As such, it is necessary to consider the change in tendon position at ultimate state in the case of external prestressing for a better prediction of the ultimate strength.

It is possible to predict the overall flexural behavior of an externally prestressed beam using a nonlinear analytical methodology consisting of a multi-level iterative technique [2]. However, this methodology is fairly complex and it is necessary to establish a simplified accurate design equation for practical situations. A design equation was proposed in a previous study considering the eccentricity variations. [2]. Based on a similar approach, a simplified modified equation was proposed incorporating the other factors that influence the ultimate tendon stress [3]. In this study, an attempt has been made to extend this equation to predict the tendon stress of continuous beams with various loading configurations.

2. BASIS OF THE PROPOSED METHODOLOGY

The general form of the ultimate tendon stress for unbonded tendons can be expressed as follows:

$$f_{ps} = f_{pe} + \Delta f_{ps} \tag{1}$$

Where f_{ps} is the ultimate tendon stress, f_{pe} is the effective initial prestress and Δf_{ps} is the increase of tendon stress. In the existing prediction equations the estimation of Δf_{ps} varies based on the equations. An equation was proposed by Naaman [4] for the prediction of ultimate tendon stress in beams with unbonded tendons, based on the concept of strain reduction coefficient Ω_u. This reduces the member dependent analysis to a simplified section dependent analysis. This equation was later adopted by AASHTO (1994) [5]. The applicability of the above equation to beams with external prestressing was carried out by Mutsuyoshi [2] and was found that it is necessary to take into account the change of tendon position at ultimate state. As a result, the concept of depth reduction factor R_d, that estimates the ultimate tendon position, was introduced and the following equation was proposed.

$$f_{ps} = f_{pe} + E_{ps}\Omega_u\varepsilon_{cu}\left(\frac{d_{pu}}{c} - 1\right) \leq f_{py} \tag{2}$$

Where E_{ps} is the modulus of elasticity, ε_{cu} is the ultimate strain of concrete at the topmost fiber, c is the neutral axis depth and f_{py} is the yield strength of prestressing steel and the ultimate tendon position d_{pu}, is given by the following expression:

$$d_{pu} \quad = \quad R_d \, d_{ps} \tag{3}$$

Where d_{ps} is the effective depth of tendon. The above equation has given the best results among all the prediction equations. However, the limitation is that it cannot be used for beams with combined prestressing consisting of internal bonded and external tendons. As such, the proposed equation was modified and a new equation was proposed to incorporate the important parameters including the influence of internal bonded tendons. The equations obtained for the strain reduction coefficient Ω_u and depth reduction factor R_d through regression analysis are expressed as follows:

(a) Strain reduction coefficient Ω_u ;

$$\Omega_u \quad = \quad \frac{2.31}{(L/d_{ps})} + 0.21\left(\frac{A_{ps,int.}}{A_{ps,tot.}}\right) + 0.06 \qquad \text{for two-point loading} \tag{4}$$

(b) Depth reduction factor R_d ;

$$R_d \quad = \quad 1.25 - 0.010\left(\frac{L}{d_{ps}}\right) - 0.38\left(\frac{S_d}{L}\right) \leq 1.0 \quad \text{for two-point loading} \tag{5}$$

in which, L is the span, S_d is the distance between the deviators, $A_{ps,int}$ is the area of bonded internal tendons and $A_{ps,tot}$ is the total area of tendons. By substituting Eqs. 4 and 5 in Eqs. 2 and 3, the expression for f_{ps} can be obtained. Considering the equilibrium of forces at the critical section, the neutral axis depth c can be computed and Eq. 2 will yield the value for f_{ps}. Once f_{ps} is known the ultimate flexural strength M_u can be calculated as explained in reference [4]. In the case of uniform loading the equation for third-point loading can be used, since the moment diagrams of these two loading patterns are approximately the same.

3. EXTENSION OF THE PROPOSED EQUATION FOR CONTINUOUS SPAN BEAMS

3.1 Concept Used in Proposed Equation

Experimental investigations [6] have shown that the increase in the ultimate tendon stress in unsymmetrically loaded continuous beams is significantly small compared to the fully loaded beams as shown in Fig. 1. This could be attributed to the smaller deflection of the lightly loaded span in the unsymmetrical loading. In a continuous beam, after the formation of the plastic hinges the whole beam could be subdivided into a series of single span beams as shown in Fig.2. The ultimate deflection of each span is dependent on the type of loading on the individual span, and which in turn influences the ultimate tendon stress of such beams. It is interesting to note that the stress increase in the symmetrically loaded two span beam is almost the same as that of

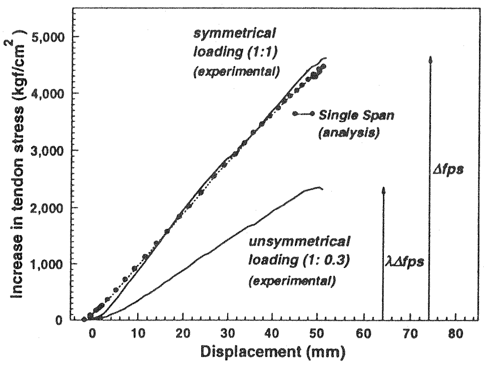
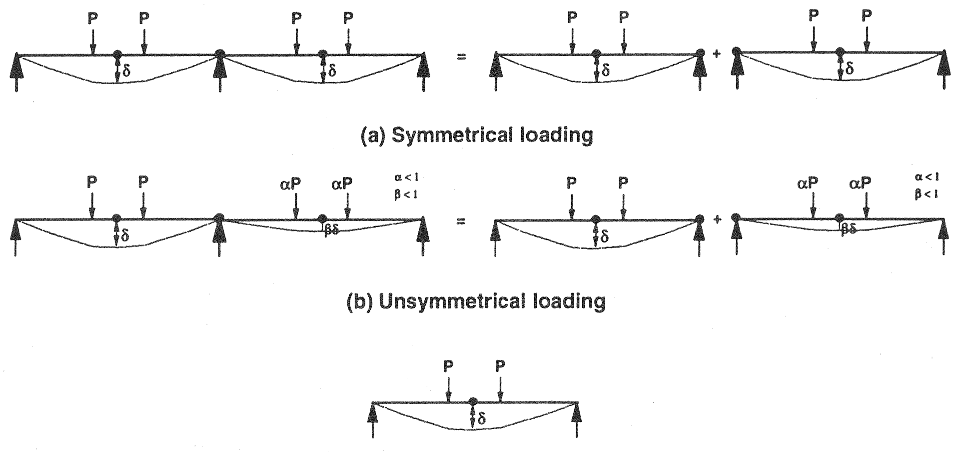


Fig. 1 Stress increase in continuous beams



(a) Symmetrical loading

(b) Unsymmetrical loading

(c) Single span beam

Fig. 2 Comparison of deflected shape of continuous beam with single span beam

the simply supported single span beam. As such, it is believed that the prediction equation for single span beams could be used for the symmetrically loaded continuous beams. However, to use the same design equation for partially loaded continuous beams, a reduction factor λ is proposed in this study to incorporate the feature of smaller stress increase.

3.2 Parametric Evaluation

Using the non-linear analytical methodology [7], the effect of partial loading on the ultimate behavior of continuous beams was studied by conducting a parametric analysis. For evaluation purpose a 2-span beam with two point loads on each span was used as shown in Fig. 3. The variables used are the span-to-depth ratio (L/d_{ps}) and the loading ratio ($L_{p,R}/L_{p,L}$). The loading on right span ($L_{p,R}$) was varied while the left span was fully loaded ($L_{p,L}$). These are summarized in Table 1. The combination of the above two variables led to a total number of 55 cases that were evaluated in this study.

Table 1. Summary of variables used in parametric evaluation

No.	Description of variables	Range	Increment	No. of cases
1	Span-to-Depth ratio (L/d_{ps})	15 - 35	5	5
2	Loading ratio ($L_{p,R}/L_{p,L}$)	0.00 - 1.00	0.1	11
			Total number of combination	**55**

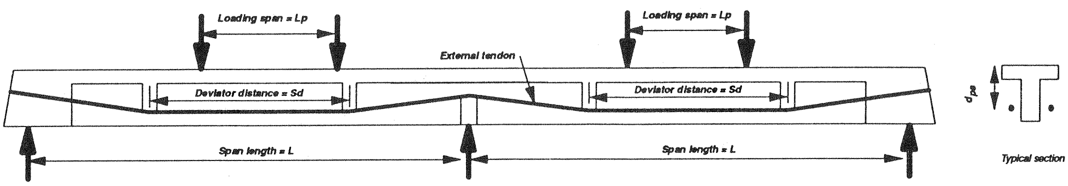


Fig. 3 Model of 2-span continuous PC beam used in the parametric evaluation

4. PROPOSED EQUATION

4.1 Reduction Factor λ

The result of the parametric evaluation is summarized in Fig. 4. It is observed that for lower load ratio, the increase in tendon stress ratio was almost negligible. However, for load ratio above 0.5 it is significant. The best fit obtained for the reduction factor λ can be expressed by the following relationship:

$$\lambda = \left(\frac{P_p}{P_u}\right)^{\alpha} \quad (6)$$

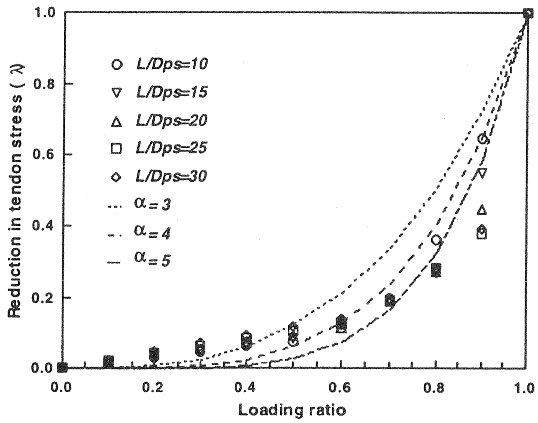


Fig. 4 Reduction in Δfps with loading ratio

Where P_p is the partially applied load and P_u is the ultimate design load in the span under consideration. In this equation α is a constant which can be assumed to be between 3 to 5. Further research is recommended to obtain the most suitable value for α. It is also possible to model the above relationship by a bi-linear or tri-linear equation.

4.2 Extension for Multi-Span Beams

For multi-span continuous beam this equation can be extended further incorporating the effect of different span lengths. The above equation can be extended by proportionally distributing the reduction factor among each span, as given below:

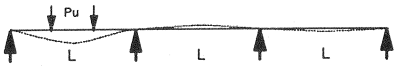
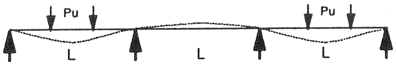
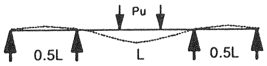
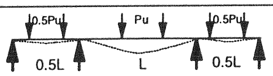
$$\lambda_n = \sum_{i=1}^{n} \frac{L_i}{L_t}\left(\frac{P_p}{P_u}\right)^{\alpha} \quad (7)$$

Where n is the total number of spans, L_i is length of the i^{th} span and L_t is the total length of the tendon between anchorage points. The reduction factor expressed in Eq. (7) is introduced in the basic expression given by Eq. (2) and the modified equation for continuous span beams shall be as follows:

$$f_{ps} = f_{pe} + \lambda_n E_{ps}\Omega_u \varepsilon_{cu}\left(\frac{d_{pu}}{c} - 1\right) \leq f_{py} \quad (8)$$

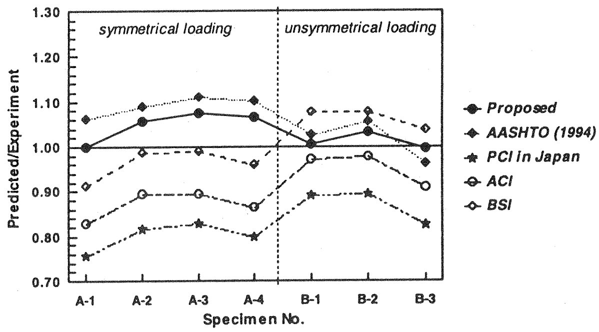
From Eq. (8), using the similar procedure of single span beams the ultimate tendon stress can be calculated as explained in [4]. In evaluating the value of reduction factor λ in a multi-span beam, it is necessary to consider the loads in each span and the length of them. Table 2 gives some typical load configuration of two span and three span continuous beams, and the corresponding value of λ depending on the loading condition. It can be seen that the value of λ increases when the number of fully loaded span is high. Once the ultimate stress in a continuous beam is obtained by the above procedure, it is possible to calculate the ultimate strength of the critical points. However, in the case of support moments the depth reduction factor R_d should be taken as unity, since there is no change in tendon position at the support due to the presence of a deviator.

Table 2. Evaluation of λ in continuous beams

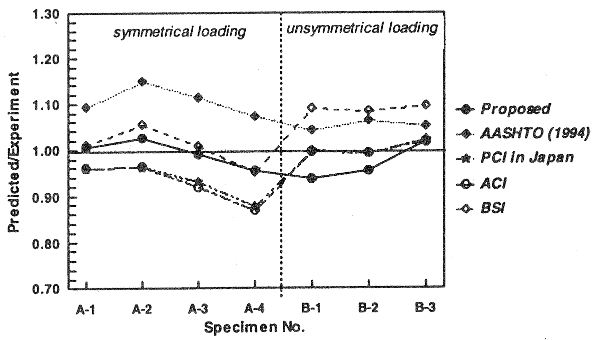
No. of span	Loading configuration	Value of λ ($\alpha = 3$)
2		$\lambda = \frac{1}{2} * (1)^3 + \frac{1}{2} * (1)^3 = 1$
		$\lambda = \frac{1}{2} * (1)^3 + \frac{1}{2} * \left(\frac{1}{2}\right)^3 = 0.563$
3		$\lambda = \frac{1}{3} * 0 + \frac{1}{3} * (1)^3 + \frac{1}{3} * 0 = 0.333$
		$\lambda = \frac{1}{3} * (1)^3 + \frac{1}{3} * 0 + \frac{1}{3} * (1)^3 = 0.667$
		$\lambda = \frac{\frac{1}{2}}{2} * 0 + \frac{1}{2} * (1)^3 + \frac{\frac{1}{2}}{2} * 0 = 0.50$
		$\lambda = \frac{\frac{1}{2}}{2} * \left(\frac{1}{2}\right)^3 + \frac{1}{2} * (1)^3 + \frac{\frac{1}{2}}{2} * \left(\frac{1}{2}\right)^3 = 0.563$

4.3 Comparison with Experimental Results

The accuracy of the proposed design equation is compared with the other design equations using the available experimental results of seven two span continuos beams with symmetrical and unsymmetrical loading as shown in Fig. 5. The accuracy of these equations is evaluated by the statistical analysis which is given in Table 3. Considering the ultimate tendon stress, the proposed equation gives an average correlation of 1.03 with the coefficient of variation (C.V) of 3%. The other prediction equations generally give a low average correlation and the coefficient of variation is also about 5%. In the prediction of ultimate strength, the proposed equation either give an average correlation of 0.98 with the C.V of 3%. It should be noted though the AASHTO (1994) equation gives a C.V of 3% the average correlation is about 9% higher than the observed values. As such it is concluded that the proposed equation predicts the ultimate tendon stress and flexural strength of continuous beams with a better accuracy.



(a) Ultimate tendon stress



(b) Ultimate strength

Fig. 5 Comparison of the accuracy of the prediction equations for continuous beams

Table 3. Evaluation of accuracy of prediction equation

Design Equation	Ultimate tendon stress		Ultimate strength	
	Average correlation	Coefficient of Variation	Average correlation	Coefficient of Variation
Proposed	1.034	2.98%	0.984	3.24%
AASHTO (1994)	1.059	4.47%	1.085	3.24%
PCI in Japan	0.832	5.44%	0.964	4.67%
ACI	0.907	5.53%	0.960	5.00%
BSI	1.007	5.59%	1.042	4.76%

5. CONCLUSIONS

A new design equation has been proposed for the prediction of the ultimate flexural strength of externally prestressed members. This equation is extended to predict the tendon stress of continuous beams with various loading configurations. The conclusions from this study are as follows.

- The span-to-depth ratio was the most important factor that affects the ultimate tendon stress in the beams with external or unbonded tendons.
- The ultimate position of the external tendon is greatly influenced by the deviator distance-to-span ratio, thus affecting the ultimate flexural strength of such beams.
- The ultimate tendon stress in partially loaded continuous beams are considerably small compared to the fully loaded beams. This is incorporated in the proposed design equation by introducing a reduction factor. Evaluation with experimental data shows good correlation.
- It is recommended that the accuracy of the proposed equation should be evaluated with other experimental investigation available for continuous PC beams.

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