Numerical Simulation Model for Hysteretic Behavior of Prestressed Concrete Box Girder

Yokohama National University, Graduate Student　○ Poologasingam NAVAKUMAR
Yokohama National University, Member of JPCEA　Tatsuya TSUBAKI

1. INTRODUCTION

Thin-walled prestressed concrete structures are frequently used in bridge structures because of its reduced self-weight and high torsional stiffness. The current design methods are insufficient to predict the ultimate behavior of a box girder bridge due to the complexity of the structural nonlinearity. Also, experimental investigations show that modeling the behavior under cyclic loading is important in the analysis of a box girder where the effect of earthquake loading is considered. Therefore, the nonlinear multi-segment model has been developed for a prestressed concrete box girder with a hollow cross section of arbitrary shape[1]. In this model, cross-sectional shape is expressed by connecting segments of different thickness. Each segment is divided into a number of layers in order to take into account the shear stress distribution and material nonlinearity along the segment.

In this paper, the load-deflection curve for a simply supported box girder is simulated for the cyclic loading in the transverse direction, considering the structural behavior for the horizontal earthquake loading. Analytical results are compared with the experimental results[2] of reinforced concrete and prestressed concrete specimens under cyclic loading. Design values for these cases[2] are also compared with the results of the multi-segment model. By calculating the shear stress distribution across the cross section, the crack pattern is plotted for the region near the loading points. Considering these results, the applicability of the present multi-segment model to a prestressed concrete box girder is discussed.

2. NONLINEAR MULTI-SEGMENT MODEL

In order to analyze prestressed concrete box type structure, the multi-segment model [1] is implemented with the material nonlinear models[3] of concrete, steel and PC tendon as shown in Fig.1. Using the Bernoulli-Euler beam theory and the engineering theory of the shear stress distribution across the cross section, the basic equations of the multi-segment model are formulated.

2.1 MATERIAL MODEL

The material model for concrete is shown in Fig.1(a). The nonlinear behavior up to the peak and the subsequent strain-softening behavior are taken into account. For steel reinforcement, the strain hardening and the Bauschinger effect are taken into account in the model as shown in Fig.1(b). A nonlinear stress-strain relationship shown in Fig.1(c) is used for PC tendon considering the behavior under cyclic loading.

(1) Concrete

The stress-strain curve of concrete is defined by Eq. (1).

$$\sigma_c = \sigma_{co}\left[2\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right) - \left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] \quad \text{(for } AB\text{);} \quad \sigma_c = \sigma_{co}\left[1 - z\left(\frac{\varepsilon_c}{\varepsilon_{co}} - 1\right)\right] \quad \text{(for } BC\text{);}$$

$$\sigma_c = 0.4\sigma_{co} \quad \text{(for } CD\text{);} \quad E_{co} = 2\frac{\sigma_{co}}{\varepsilon_{co}}; \quad z = \frac{1}{3+\rho_w}\left(0.0016\sigma_{co} - 0.09\right) \qquad (1)$$

where $\sigma_{co}, \varepsilon_{co}$ are defined at the point B and ρ_w is the area ratio of shear reinforcement for confinement in the transverse direction.

(2) Steel

The stress-strain curve of the GMP model [3] is used for steel reinforcement as defined by Eq.(2).

$$\sigma_s^* = b\sigma_s + (1-b)\frac{\varepsilon_s^*}{(1+\varepsilon_s^{*R})^{\frac{1}{R}}};\quad \varepsilon_s^* = \frac{\varepsilon_s-\varepsilon_{sr}}{\varepsilon_{so}-\varepsilon_{sr}};\quad \sigma_s^* = \frac{\sigma_s-\sigma_{sr}}{\sigma_{so}-\sigma_{sr}};$$

$$b = \frac{E_{s1}}{E_{so}};\quad R = R_o\text{-}8.5\frac{\xi}{0.5+\xi} \qquad (2)$$

Where $\sigma_s^*, \varepsilon_s^*$ stand for the equivalent stress and strain respectively. b is a parameter for the hardening effect, R and R_o are for the Bauschinger effect,$(\sigma_{sr},\varepsilon_{sr})$ are of the values at the points C and E, and $(\sigma_{so},\varepsilon_{so})$ are of the values at the points B, D and F. E_{so} and E_{s1} represents the initial slope and the secondary slope. ξ is obtained as the difference between the absolute value of maximum experienced strain and the absolute value of the yield strain, divided by the yield strain.

(3) PC Tendon

The stress-strain curve for the PC tendon shown in Fig. 1(c) is expressed by Eq. (3).

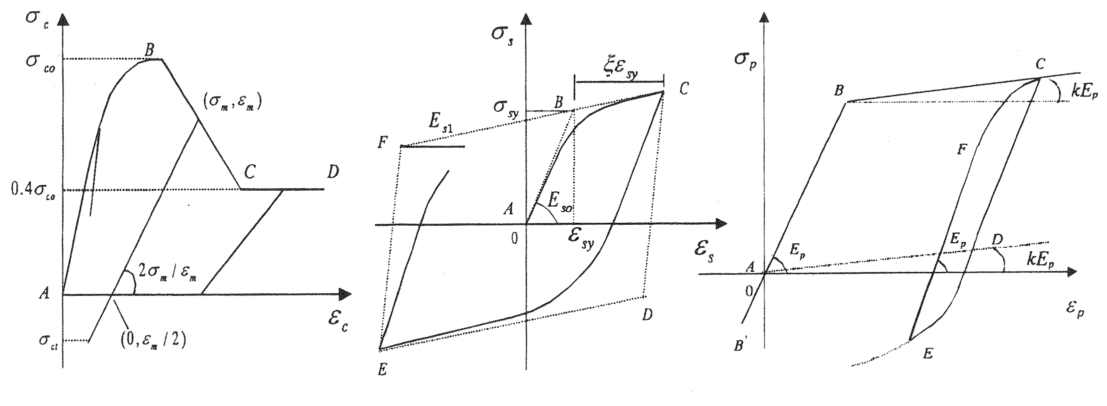
$$\sigma_p = E_p\varepsilon_p \text{ (for } B'B\text{)};\quad \sigma_p = \sigma_{py} + kE_p(\varepsilon_p - \varepsilon_{py}) \text{ (for } BC\text{)};\quad \sigma_p = \sigma_{pm}\text{-}E_p(\varepsilon_{pm}\text{-}\varepsilon_p) \text{ (for } CD\text{)};$$

$$\sigma_p = \sigma_{pm} - \sigma_{py} + \frac{E_p(\varepsilon_p - \varepsilon_{pm} + \varepsilon_{py})}{1-\left(\frac{E_p}{\sigma_{pm}} - \frac{1}{\varepsilon_{pm}}\right)(\varepsilon_p - \varepsilon_{pm} + \varepsilon_{py})} \quad \text{(for } DE\text{)};$$

$$\sigma_p = \sigma_{pn} + E_p(\varepsilon_p - \varepsilon_{pn}) \quad \text{(for } EF\text{)};$$

$$\sigma_p = \sigma_{pn} + \sigma_{py} + \frac{E_p(\varepsilon_p - \varepsilon_{pn} - \varepsilon_{py})}{1+\left(\frac{E_p}{\sigma_{pm} - \sigma_{pn} - \sigma_{py}} - \frac{1}{\varepsilon_{pm} - \varepsilon_{pn} - \varepsilon_{py}}\right)(\varepsilon_p - \varepsilon_{pn} - \varepsilon_{py})} \quad \text{(for } FC\text{)} \qquad (3)$$

where $(\sigma_{py},\varepsilon_{py})$, $(\sigma_{pm},\varepsilon_{pm})$ and $(\sigma_{pn},\varepsilon_{pn})$ are defined at the points B, C and E respectively.



(a) Concrete　　(b) Steel　　(c) PC tendon

Fig. 1 Material models used in the analysis

2.2 MULTI-SEGMENT MODEL

In order to analyze the box girder bridge, the formulation is extended from the three-dimensional layered beam element model [4] for a solid cross section. From the normal stress acting on the cross section in the longitudinal direction, the shear stress is obtained using the stress equilibrium and the continuity condition of the closed section. The shear flow across the cross section is obtained as shown in Eq. (4)[1].

$$q_s = \frac{-\oint_{s_{cl}}\left(\int_{s_p}^{s}(Ey\frac{d^3v}{dx^3}+Ez\frac{d^3w}{dx^3})tds\right)\frac{ds}{t}}{\oint_{s_{cl}}\frac{ds}{t}} + \int_{s_p}^{s}(Ey\frac{d^3v}{dx^3}+Ez\frac{d^3w}{dx^3})tds \quad (4)$$

where t_{ij} is the thickness of segment, l_{ij} is the length of segment, q_s is the shear flow and s is the distance along the segment. u, v, w are the displacements at the centroid in the x, y and z direction respectively and E is the Young's modulus of concrete.

Substituting the displacement-nodal displacement relationship for v and w into Eq.(4), the shear flow q_s can be written in a matrix form with the nodal displacements. The interpolation functions for v and w are cubic polynomials while that for u is a linear function. The shear stress-strain relationship is given by Eq. (5). The shear strain-nodal displacement relationship is given in the matrix form as shown in Eq. (6).

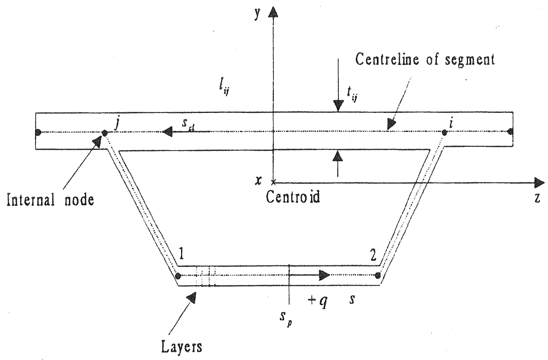


Fig. 2 Geometry of box type cross section for the multi-segment model

$$\frac{q_s}{t} = G\gamma \quad (5)$$

$$\gamma = B_s u^e \quad (6)$$

$$u^e = [u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \theta_{z1}, u_2, v_2, w_2, \theta_{x2}, \theta_{y2}, \theta_{z2}]^T \quad (7)$$

where u^e is the nodal displacement vector in local x, y and z directions as given in Eq. (7). The shear strain –nodal displacement matrix B_s is obtained from Eq.(4) and it depends on the cross-sectional shape with nodal displacement components and the material property. γ stands for the shear strain and G is the shear modulus of concrete.

2.3 COMPUTATIONAL PROCEDURE

The computational procedure of the present analysis method using the multi-segment model is summarized as follows.

(1) The strain at each point of segment used for the present load step is calculated by the previous values of strain and strain increment using the Euler method.

(2) The tangential modulus is calculated for each material from its stress-strain relationship.

(3) The global incremental stiffness eaquation is obtained by assembling the element incremental stiffness equation given by Eq.(8), where f^e is the nodal incremental load and f^{ep} is the initial load due to prestress transformed from the tendon point to the centroid of the cross section.
(4) Solving the global incremental stiffness equation under displacement control, the unknown nodal displacements are obtained to calculate the strain and stress at each point.
(5) Repeat the above procedure until the specified load level reached.

$$k^e \Delta u^e = (\Delta f^e + \Delta f^{ep}) \tag{8}$$

$$f^e = \left[f_{x1}, f_{y1}, f_{z1}, m_{x1}, m_{y1}, m_{z1}, f_{x2}, f_{y2}, f_{z2}, m_{x2}, m_{y2}, m_{z2}\right];$$

$$f^{ep} = \left[f^p_{x1}, f^p_{y1}, f^p_{z1}, m^p_{x1}, m^p_{y1}, m^p_{z1}, f^p_{x2}, f^p_{y2}, f^p_{z2}, m^p_{x2}, m^p_{y2}, m^p_{z2}\right] \tag{9}$$

3. NUMERICAL SIMULATION

During the earthquake motion a bridge structure may experience transverse cyclic loading. In this case, if there is a restraint between the girder and the support, it is important to consider the behavior of the girder part of the bridge structure in the transverse direction. The experimental results [2] show the hysteretic behavior of the box type structure and it is simulated by the multi-segment model for the reinforced and prestressed concrete box girder.

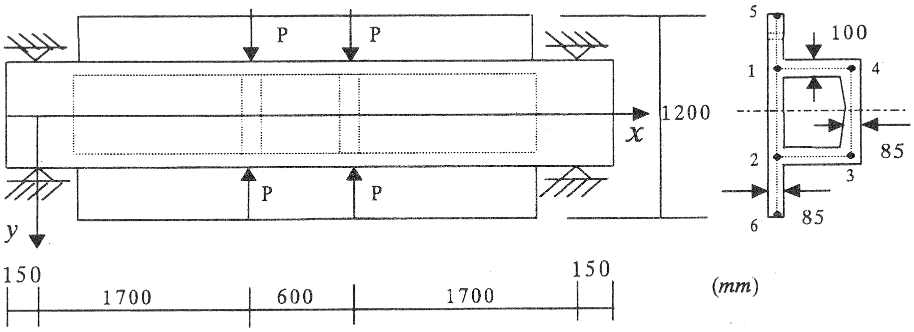


Fig. 3 Geometry and modeling of the simply supported box girder loaded in the transverse direction

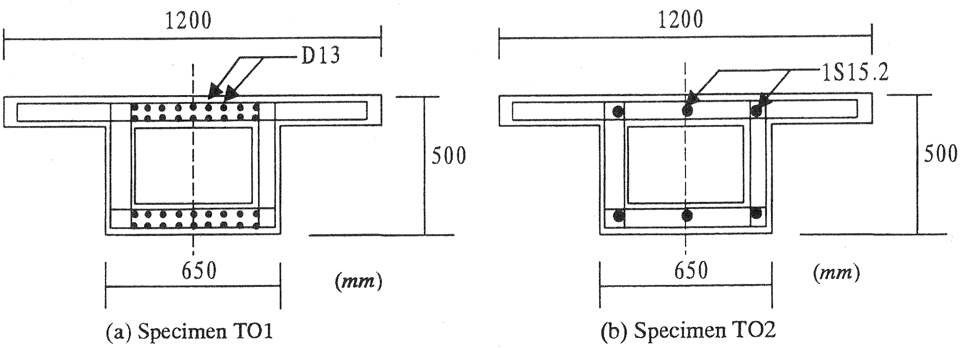
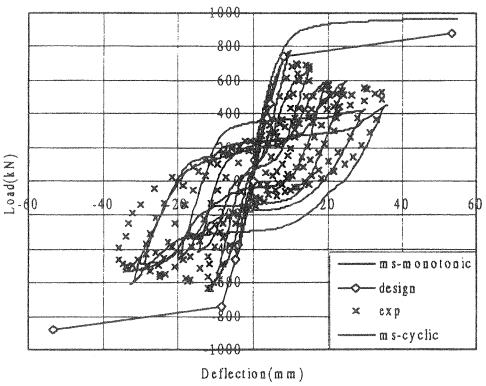


Fig. 4 Geometry of the cross-sectional shape

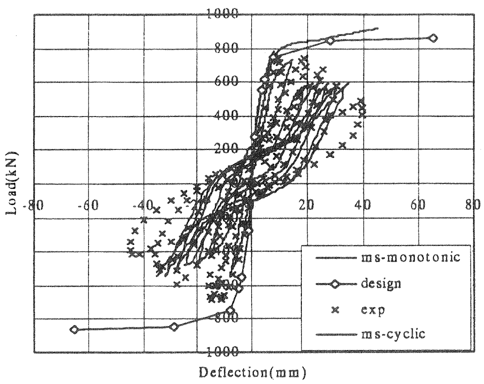
A simply supported symmetrically loaded box girder beam shown in Fig.3 is modeled using the multi-segment model to analyze the deformational behavior under cyclic loading in the transverse direction. Specimen TO1 of reinforced concrete and specimen TO2 of prestressed concrete are analyzed. Concrete strength is 40 N/mm^2. The steel reinforcement is D13 (SD295) and the PC tendon is 1S15.2 [2].

The cross section is divided into six segments by connecting internal nodes as shown in Fig.3 along the centerline of the segment. Each segment in the flange part is divided into twenty layers of the same size to take into account the material nonlinearity and the shear stress distribution along the segment. Considering the half part of the simply supported box girder, it is divided into twenty-four elements. Eight smaller size element of 50 mm near the loading point and sixteen 100 mm elements away from the loading points are used in the analysis. In addition to the reinforcement shown for the specimen TO1 and TO2, D6 steel bars are added as the longitudinal and shear reinforcements[2]. In specimen TO2, six PC tendons of 1S15.2 are used as shown in Fig.4(b).



ms : multi-segment model
exp: experiment

Fig.5 Load -deflection curve for specimen TO1



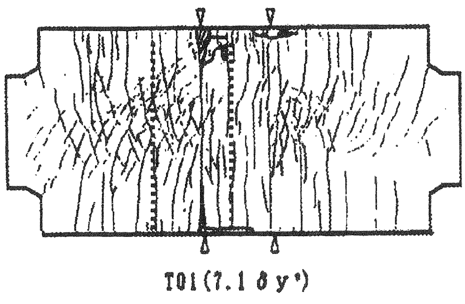
ms : multi-segment model
exp: experiment

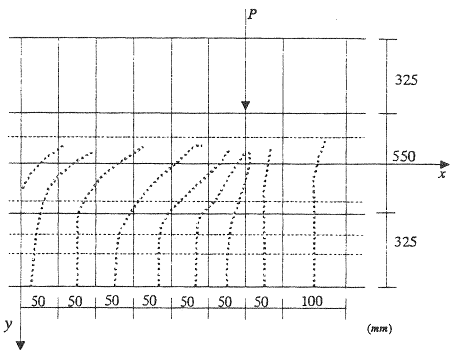
Fig.6 Load -deflection curve for specimen TO2

The design values of loads[2] for cracking, first reinforcement yielding, web reinforcement yielding, concrete failure in compression are compared with the analysis results of the monotonic load-deflection curve shown in Fig.5 and Fig.6.

The behavior under cyclic loading in the transverse direction is simulated by the multi-segment model for specimens TO1 and TO2 as shown in Fig.5 and Fig.6 respectively. The energy dissipation is higher for the cyclic loading of specimen TO1 compared to that of specimen TO2 from the experimental and analytical results using the multi-segment model, as seen from Fig.5 and Fig.6 respectively.

Experimental investigation [2] also shows the crack pattern under cyclic loading as shown in Fig.7(a). Using the multi-segment model, the principal plane and the direction of cracking are calculated for each layer of a segment under monotonic loading. The crack pattern obtained from the analysis for the domain between dashed lines in Fig.7(a) is shown in Fig.7 (b). The analysis result of the crack pattern near the loading point under monotonic loading upto failure is plotted. It is observed that the analysis result for the crack pattern is in good agreement with the test result.





(a) Test result

(b) Analysis result

Fig. 7 Crack pattern for the specimen TO1

4. CONCLUSIONS

A numerical simulation model is presented for the analysis of a prestressed concrete box girder under cyclic loading. Conclusions in this study are summarized as follows.

(1) The multi-segment model is suitable for the analysis of a prestressed concrete girder with a hollow cross section to take into account the shear stress distribution more accurately.

(2) The deformational behavior under cyclic loading is properly analyzed by using the hereditary material models for concrete, steel reinforcement and PC tendon.

REFERENCES

1) Navakumar, P. and Tsubaki, T.: Nonlinear Analysis of Prestressed Concrete Box Girder with Multi-segment Model, The Third International Summer Symposium, JSCE, 2001 (in print).

2) Okada, T. et al.: Hysteresis Characteristics of PC Box Girders in Transversal Direction due to the Cyclic Loading Test, Proceedings of the 10th Symposium on Developments in Prestressed Concrete, JPCEA, pp. 527-532, 2000.

3) Li, K., Otami, S. and Aoyama, H.: Behavior of R/C Column Under Varying Axial and Earthquake Loads, Proc., JCI 8th Conference, pp. 489-492, 1986.

4) Tamura, T. and Tsubaki, T.: Three-Dimensional Analysis for Concrete Members Using Layered Beam Elements, Proceeding of the 9th Symposium on Developments in Prestressed Concrete, JPCEA, pp. 1-6, 1999.