INFLUENCE OF NON-PRESTRESSED REINFORCEMENT ON BEHAVIOR OF EXTERNALLY PRESTRESSED CONCRETE BEAMS

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1. INTRODUCTION.

In an external prestressing system, the prestressing cables are unbonded to the concrete beam because they are located outside the cross section. Therefore, the same parameters that are known to influence the behavior of beams with internally unbonded cables are expected to influence beams with external cables, such as reinforcement ratio, span-to-depth ratio, cable profile, loading pattern, etc. It is experimentally agreed that the ultimate strength of prestressed concrete beams with unbonded cables or external cables is comparatively smaller than that of the similar prestressed concrete beams with internally bonded cables, the difference being placed at 10-30% [6]. The reasons for the lower strength of beams with unbonded cables can be explained that since the cable is generally free to slip, the strain in the cable is more or less equalized along its length, and the strain at the critical section is lessened, leading to the lower strength of beams. Moreover, the beams with unbonded cables tend to develop a few large cracks in the vicinity of the critical sections instead of many small ones well distributed. Such these cracks tend to concentrate the strain in the concrete at these sections, and rapidly increase in width and depth as the load increases, thus resulting in premature failure.

To avoid the undesirability of such behavior in the beams prestressed with unbonded cables, non-prestressed reinforcement is commonly added to help overcoming the formation of sparsely spaced wide cracks and the concentration of compressive strain above these cracks. Such addition is attributed to the resistance of the non-prestressed reinforcement itself as well as to its effect in distributing and limiting the cracks in the concrete, help carrying the tensile stresses in the concrete, leading to give higher strength to the beam. Actually, the addition of non-prestressed reinforcement serves two main purposes: 1) to well distribute the crack along the beam length 2) to contribute to the ultimate load capacity of the beam.

In this study, a nonlinear analysis of the flanged beams prestressed by means of external cables with various amount of non-prestressed reinforcement was performed. A comparison of load-displacement relationship was then discussed with emphasis on the effect of non-prestressed reinforcement. The validity of the proposed method for externally prestressed concrete beams was verified by comparing the predicted results in terms of moment vs. deflection with experimental data.

2. NON-LINEAR ANALYSIS ALGORITHM AND STRAIN VARIATION OF CABLE

2.1 Non-Linear Analysis Algorithm

In order to obtain a whole deformed shape of beam, a finite element method is commonly used as one of powerful and popular tools in the structural analysis. The conventional finite element method often approximates a deformed shape of beam element with interpolation functions such as a cubic polynomial function for transverse displacement and a linear function for longitudinal displacement. The cubic function implies a linear variation of curvature along the element. However, the analysis of unbonded beams in general or the analysis of externally prestressed concrete beams in particular necessitates an accurate evaluation of strain variation in the concrete since the compatibility equation should be formulated with the values of concrete strain at the lever of cable. Thus, a large number of short elements are necessary for the adequate evaluation of cable strain.

In the previous study, a non-linear finite element program together with the displacement control method had been developed to obtain the entire behavior of externally prestressed concrete beam up to the ultimate limit state [4]. The program used a stepwise analysis and deformation control to trace the nonlinear response of prestressed concrete beams with external cables. The program is capable of accounting for not only the flexural deformation, but also for the shear deformation, friction at the deviators, and external cables with different configuration (straight or polygonal profile). In the analysis, the beam was represented by a set of beam elements connected together by nodes located at either end. Each node has three degrees of freedom, namely, horizontal displacement, vertical displacement and rotation. A cable stress equal to the effective stress after all losses, was taken as the initial value in the analysis. Cross section of the beam was divided into layers, in which each layer might have different materials, but its properties were assumed to be constant over the layer thickness. Based on the effective stress of cable, the concrete strain of each layer for every beam element was determined, and appeared to take as the initial condition of beam. In this study, the only one displacement control point, which could be arbitrarily chosen among the points of the applied load, was applied in the analysis. The analytical flowchart was presented in **Fig.1**.

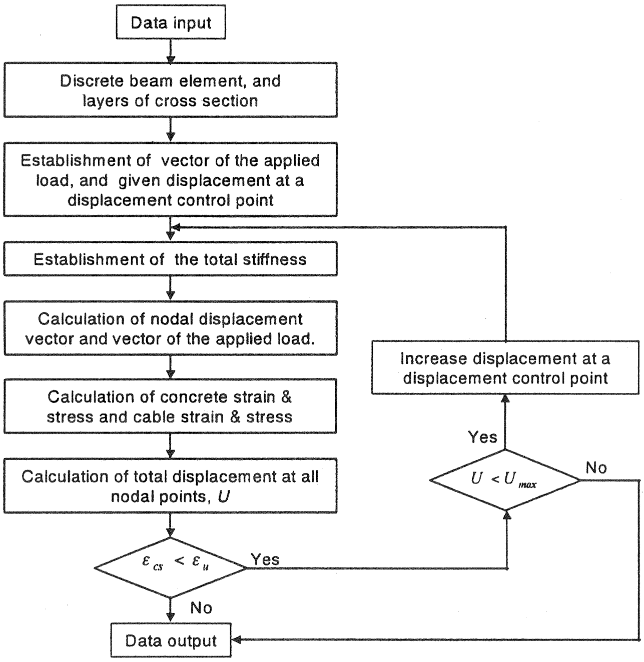


Fig.1 Flowchart of analysis

2.2 Strain Variation in External Cable

Since the deflection of external cables does not follow the beam deflection except at the deviator points, the strain in a cable totally differs the strain in the concrete at the cable level. Therefore, the cable strain cannot be determined from the local compatibility of deformation. When the beams are subjected to bending, the strain induced in the concrete at the cable level varies according to the bending moment diagram. Therefore, an analytical model should be satisfied the total compatibility requirement that the total elongation of an external cable must be equal to the integrated value of concrete deformations at the level of cable. In the previous study [4], when structural behavior of externally prestressed concrete beams was investigated, an analytical approach is usually based on the assumption, which is referred to as "*Deformation Compatibility of Beam* , i.e., the total elongation of cable must be equal to the total elongation of concrete at the cable level between the extreme ends. This can be expressed in the following equation:

$$\sum_{i=1}^{n} l_i \Delta\varepsilon_{si} = \int_{o}^{l} \Delta\varepsilon_{cs} dx \qquad (1)$$

where $\Delta\varepsilon_{si}$,$\Delta\varepsilon_{cs}$ are the increments of cable strain and concrete strain at the cable level, respectively; l_i is the length of cable element under consideration; and l is the total length of cable between the extreme ends.

3. NUMERICAL EXAMPLES

A total of six prestressed concrete beams with external cables, which had been tested by Zhang Z., et al., [8], were considered to analyze as numerical examples in this study. All the beams were the simply supported beams with a flanged section, and were divided into two groups with three beams of each. In each group, the beams were designed with different amount of non-prestressed reinforcement in order to examine its effect on the behavior of the prestressed beams with external cables at ultimate. The beams in the first group were prestressed by two cables with the straight profile at the depth of 350.0 mm from the top surface of the beam (beams with series A). While in the second group, the beams were prestressed by two cables with the polygonal profile at the depth of 375.0 mm, and two deviator points were provided at the distance of 2700.0 mm (beams with series B). All the beams were subjected to two points of the external load located at the distance of 1000.0 mm with symmetrical loading arrangement. **Fig.2** showed the layout scheme of representative beam of each group. Material properties and test variables were shown in **Table 1**. A more detailed test setup and geometrical dimensions of the beams can be found elsewhere [8].

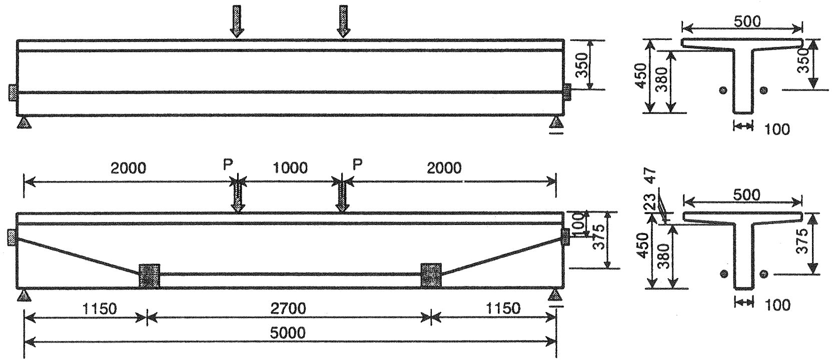


Fig.2 Layout scheme of simply supported beams with external cables

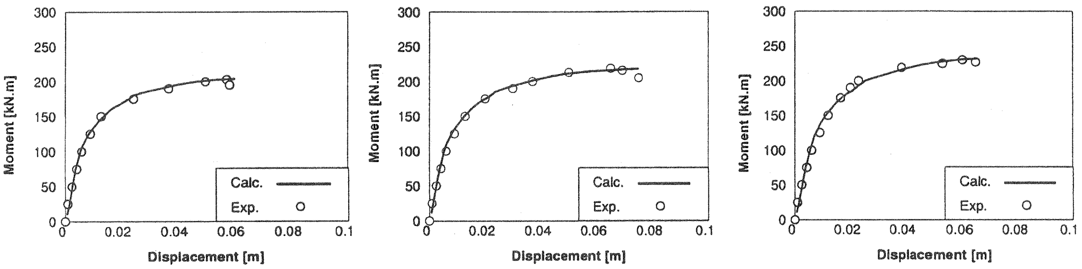
Table 1 The tested beam variables and their materials

No	A_s mm^2	f_y Mpa	d_s mm	A_p mm^2	f_{pe} Mpa	d_p mm	f'_c Mpa	q_s	q_{pe}	q_o
A1	157.0	267.0	410.0	981.7	325.9	350.0	52.3	0.0039	0.0350	0.0389
A2	235.6	267.0	370.0	981.7	335.4	350.0	49.8	0.0068	0.0378	0.0446
A3	358.2	267.0	383.7	981.7	326.8	350.0	52.6	0.0095	0.0350	0.0445
B1	157.0	267.0	390.0	392.7	805.7	375.0	52.7	0.0041	0.0320	0.0361
B2	201.1	267.0	410.0	392.7	843.4	375.0	52.7	0.0050	0.0335	0.0385
B3	402.1	267.0	390.0	392.7	822.2	375.0	49.3	0.0112	0.0350	0.0462

Notations are defined as in Eq.(2)

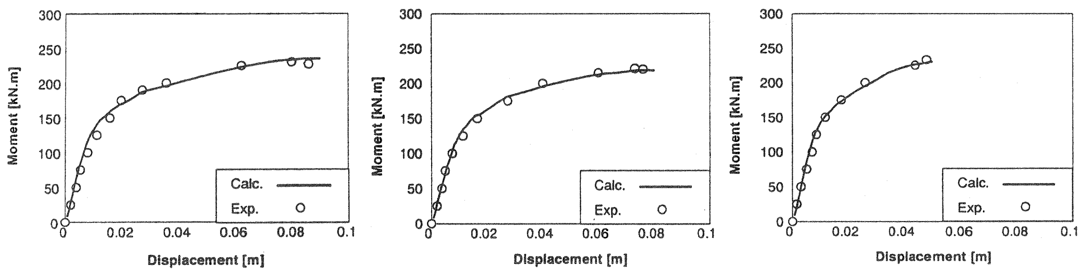
3.1 Discussion of Analytical Results

Fig.3 and **Fig.4** showed the moment-displacement relationships of beams with series A and B, respectively. It can be seen from these figures that the predicted moment-displacement responses are agreed well with experiment data for the beams with different configuration of external cable. At the ultimate state, the predicted maximum moments were about of 203.9 kN.m, 218.0 kN.m, 231.8 kN.m, 234.6 kN.m, 218.4 kN.m and 229.7 kN.m, corresponding to the maximum deflections of 60.4 mm, 75.0 mm, 65.0 mm, 90.2 mm, 80.1 mm and 50.0 mm for the beams A1, A2, A3, B1, B2 and B3, respectively. The difference in the applied moment of beams could be attributed to the additional amount of non-prestressed reinforcement. Moreover, the first crack of each beam occurred at the different level of applied moments, which were approximately 94.3 kN.m, 96.7 kN.m, 104.4 kN.m, 105.0 kN.m, 113.5 kN.m and 114.6 kN.m for the beams A1, A2, A3, B1, B2 and B3, respectively. It is apparently shown that the ultimate moment capacity of a beam will be higher when the first crack occurs at a higher applied moment, and it will be lower when the first crack occurs at a lower applied moment. After cracking, the beam with smaller amount of non-prestressed reinforcement went through much larger curvatures before reaching ultimate moment capacity than did its companion beams with larger amount of non- prestressed reinforcement. The analytical results reproduced the experimental data with remarkably good agreement, and the method of prediction can accurately show the general behavior of prestressed concrete beams with external cables.



a) Beam A1　　b) Beam A2　　c) Beam A3

Fig.3 Moment-displacement relationships of beams with series A

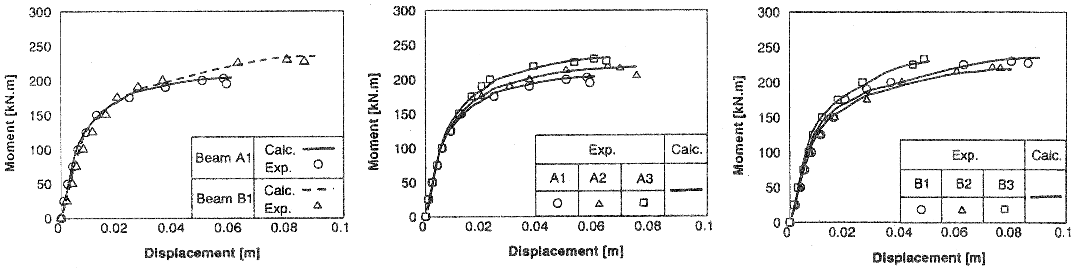


a) Beam B1　　b) Beam B2　　c) Beam B3

Fig.4 Moment-displacement relationships of beams with series B

3.2 Effect of Cable Configuration

The effect of cable configuration was examined by comparing the obtained results of the beam A1 with the beam B1. The beam A1 was identical to the beam B1 except the cable area and the cable configuration. Since both beams A1 and B1 had essentially the same prestressing force at the initial stage, the effective stress of cable was significantly different because of using the different amount of cable area (981.7 mm^2 and 325.9 N/mm^2 of the beam A1 compared to 392.7 mm^2 and 805.7 N/mm^2 of the beam B1). It can be seen from the **Fig.5a** that the moment-displacement responses behaved essentially similar up to the decompression, indicating that the cable configuration had no effect on the displacement response. The beam B1, however, registered a higher ultimate strength and a larger deflection than that of the beam A1 due to a larger eccentricity of cable (234.6 kN.m and 90.2 mm compared to 203.9 kN.m and 60.4 mm). This difference in the ultimate strength could also be attributed to the presence of deviator points. The beam A1 had no deviator, while the beam B1 had two deviators located symmetrically at distance of 1350.0 mm from the center of beam. That is, the beam B1 had shortly free length of cable, leading to a higher increase of cable stress at ultimate, and consequently, in result of a higher ultimate strength. Moreover, the beam B1 had a bigger camber due to a larger eccentricity of cable than that of the beam A1 at the prestressing stage (the cambers of the beams did not show in **Fig.5a**), i.e., the beam B1 was comparatively prestressed more. As a result, the ultimate strength of the B1 was approximately 15% higher than that of the beam A1.



a) Beams A1-B1　　b) Beams with series A　　c) Beams with series B

Fig.5 Effect of non-prestressed reinforcement and cable configuration

3.3 Effect of Non-Prestressed Reinforcement

Fig.5b showed a comparison of moment-displacement responses for the beams with series A. It can be seen in this figure that since the stiffness of the beams prior to cracking remained the same, all the beams exhibited essentially identical up to the decompression, indicating no effect resulted in using the different area of non-prestressed reinforcement. However, the first crack of the beam with a smaller amount of non-prestressed reinforcement occurred a little earlier than does the companion beams with a larger amount of reinforcement as mentioned above. Consequently, the ultimate moment of this beam was smaller than that of the beams with a higher amount of non-prestressed reinforcement. After cracking, the beam with a smaller amount of non-prestressed reinforcement exhibited a rather ductile, went through much large deflection before reaching the ultimate strength capacity, and failed by initial yielding of non-prestressed reinforcement, sequentially, collapsed totally by crushing of concrete at the compression region. On the other hand, the beam with a higher value of non-prestressed reinforcement exhibited rather stiff, resulting in a higher strength at ultimate, and generally failed at relatively small deflection.

It is well known from theory that a section of beam begins to crack whenever the applied moment on this section exceeds the cracking moment. From the analytical results, it is interesting to note that the cracks mostly concentrate within the constant moment region between the loading points. The location of cracks spread in a wider area in the beam with a higher value of non-prestressed reinforcement. And the cracks tend to appear beyond the constant moment at the ultimate state as the amount of non-prestressed reinforcement increases. Although the experiment did not show the crack pattern of the tested beams, it is, however, apparently believed that the predicted results were basically identical to the observed ones. The same observations were obtained from the tests of partially prestressed concrete beams with unbonded cables, which have been reported elsewhere [2,5]. It is also seen from this figure that the ultimate moment of the beams increased with increasing the amount of non-prestressed reinforcement. The same behavior for the beams with polygonal cable profile was obvious as shown in **Fig.5c**.

It is also proved by experiments that the ultimate strength of unbonded beams depends not only on the amount of non-prestressed reinforcement, but also depends on the amount of prestressing cable. A combination between the non-prestressed reinforcement and the prestressing cable is characterizied by a reinforcement index q_o, and is defined as:

$$q_o = q_s + q_{pe}$$

$$q_o = \frac{A_s f_y}{bd_s f_c'} + \frac{A_p f_{pe}}{bd_p f_c'} \tag{2}$$

where q_s, q_{pe} are the index of non-prestressed reinforcement and prestressing cable, respectively; A_s is the area of non-prestressed reinforcement; A_p is the area of prestressing cable; f_y is the yield strength of non-prestressed reinforcement; f_{pe} is the effective prestress of prestressing cable; f'_c is the compressive strength of concrete; b is the width of the compressive face; d_s is effective depth of beam to centroid of non-prestressed reinforcement; d_p is the effective depth of beam to centroid of prestressing cable. The values of q_s, q_{pe} and q_o given in **Table 1** reflected the actual material properties of each beam. Since the value of q_{pe} was essentially identical to all beams of each group, the main difference in ultimate strength of the beams was attributed to the index of non-prestressed reinforcement. From this point of view, it could be said that the adequate addition of non-prestresed reinforcement can be substantially enhanced the ultimate strength of the beams prestressed by external cables. The beneficial effect of non-prestressed reinforcement was also found by test of the partially prestressed beams with unbonded cables, which have been reported elsewhere [2,5,7,8]. It should be noted that in practice, it is commonly referred to the lightly reinforced beam as underreinforced and it exhibits a ductile failure. As the amount of non-prestressed reinforcement is increased, the beam approaches the overreinforced case. The overreinforced beam would exhibit crushing of the concrete with the reinforcement in the elastic range resulting in a brittle failure. This type of the beam, however, should be avoided in order to prevent the sudden collapse of the beams in design practice.

4. CONCLUSIONS

The close agreement between the experimental data and the analytical results apparently indicates the feasibility of the proposed method for the analysis of prestressed concrete beams with external cables. The ultimate strength of externally prestressed concrete beams can enhance by the adequate addition of bonded non-prestressed reinfocement. The effect of non-prestressed reinforcement on the behavior of the beam prestressed with external cables should be properly taken into consideration in design practice.

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