報告

The Fundamental Study on the Electrical Analogs of Prestressed Concrete Structures

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1. Synopsis

Recently the prestressed concrete (PC) engineering has made a remarkable progress. However, some engineers have the tendency to think that the design of PC structures is strange and rather complicated.

PC structures are seemed to be misunderstood by them. This is attributed to their not understanding that the prestressing force can be replaced by the external force working on the concrete.

Regarding PC beam as the ordinary concrete beam to which the external force corresponding to the prestressing one is applied, the analysis of prestressed concrete structures can be intelligently accomplished.

The external force to be treated instead of the prestressing one is found for the cases that prestressing tendons are located in the straight and in the parabolic.

Furthermore the decrease of tensile stresses of the tendons due to friction between the tendons and materials around this ones (e.g. concrete, sheath, etc.) is also computed herein.

In general, it is not easy to obtain the indeterminate force which occurs in the statically indeterminate structures. Besides in the PC structures, the secondary stress produced by prestressing the tendons having to be considered, the design of PC structures becomes to be more difficult.

From this point of view, the authors have tried to find the analogous circuits equivalent to some statically indeterminate PC beams and by using the analogous ones have solved them. The present paper reports the results of this fundamental study.

2. The equivalent loads for the basic locations of PC tendons

At first for the plain example, it is treated with the case that PC tendons are located in the centroid of the cross section of PC rectangular beams and are tensioned by the prestressing force P working on the beams. Of cource, in this case the prestress becomes to be uniform across the section.

Hence the compressive stress σ_c produced in concrete is expressed in the following equation with P and A (the cross sectional area of concrete).

$$\sigma_c=P/A \quad\cdots\cdots(1)$$

The bending moment M acting on the beams to which the prestress is introduced, the stress of concrete σ_c is given as follows,

$$\sigma_c=\frac{P}{A}+\frac{M\cdot y}{I} \quad\cdots\cdots(2)$$

where, I is the moment of inertia with respect to the centroidal axis of a cross section, y is the distance from the centroid of a cross section to an optional point and the positive value of y is of upward and the negative one is of downword.

As PC tendons are located e (eccentric distance) apart from the centroidal axis, the effectiveness of the prestressing force is considered to be devided into two parts, the one of which is concerning to the normal force working at the

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centroidal axis of cross section and the another is to the bending moment $P \cdot e$. The eccentric distance e is negative downwards the centroidal axis of a cross section and is positive upwards the centroidal axis of a very cross section. After all, when the external bending moment M acts on the beam, the stress σ_c produced in the beam is expressed in the equation 3.

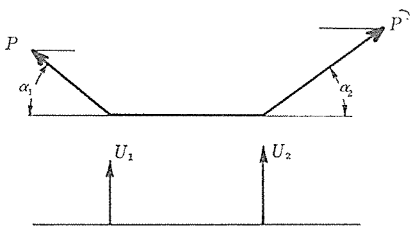
$$\sigma_c = \frac{P}{A} + \frac{P \cdot e \cdot y}{I} + \frac{M \cdot y}{I} \quad \cdots\cdots (3)$$

By the way, the P C tendon being located in the bent or in the curved and the axis of concrete members being of curved, the analysis of P C structures becomes a little more complex. The treatment of these cases are described as below.

(1) In the first case, P C tendon is located in the bend.

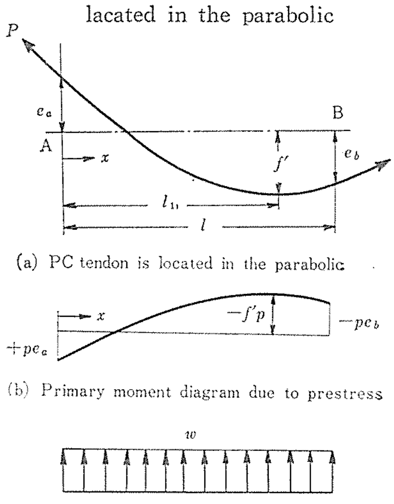
In this case, when the tensioning force of P C tendon is P, the equivalent forces acting on the concrete are the upward concentrating loads U_1 and U_2 as shown in **Fig. 1**. They are expressed in the following equations.

Fig. 1 The case that P C tendon is located in the bent



Loading diagram

Fig. 2 The case that P C tendon is lacated in the parabolic



(a) PC tendon is located in the parabolic

(b) Primary moment diagram due to prestress

(c) Loading diagram

$$U_1 = P \tan \alpha_1, \quad U_2 = P \tan \alpha_2 \quad \cdots\cdots (4)$$

However in this case, the degree of the bent of P C tendon is presumed to be very small. And the tensioning force is presumed to be constant because of that the span is so large.

(2) In the second case, P C tendon is located in the parabolic

As shown in **Fig. 2**, the anchoring points of P C tendon are different in height at the both ends of a beam. Here the P C tendon is presumed to be located in the parabolic and flat comparatively and the tensioning force is presumed to be constant at any section of the tendon. The inner bending moment M_x is due to the prestressing force is given below.

$$\left.\begin{aligned} M_x &= a\,x^2 + bx + c \\ a &= P \cdot \frac{(el_1 - fl)}{(l_1^2 l - l_1 l^2)} \\ b &= P \cdot \frac{(fl^2 - el_1^2)}{(l_1^2 l - l_1 l^2)} \\ f &= e_a + f' \\ e &= e_a + e_b \end{aligned}\right\} \quad \cdots\cdots (5)$$

where,

e_a, e_b : eccentric distance of PC tendon at the both ends of a beam

l : span length

f' : eccentric distance of PC tendon apart from the beam end

c : constant

P : tensioning force

Now differentiating M_x with respect to x, we have

$$\frac{d^2 M_x}{dx^2} = Q_x = 2\,a\,x + b \quad \cdots\cdots (6)$$

where, Q_x : shearing force.

And differentiating this equation once more, we have

$$\frac{d_2 M_x}{dx} = 2\,a = 2 \cdot P \cdot \frac{(el_1 - lf)}{l_1 l (l_1 - l)} \quad \cdots\cdots (7)$$

This makes it clear that such a P C beam as mentioned above is understood to be subjected to the upward uniform load.

(3) In the third case, the decrease of the tensile stress of P C tendon due to friction is considered

Actually in P C structures, the tensile stress of P C tendon decreases gradually from a tensioning end by the friction between PC tendon and concrete around them. So in the case that the frictional loss is to be considered, a little modification must be performed in the calculations of the equation (4) to (7), inclusive.

The tensile stress of PC tendon at the section apart L from the tensioning end is represented.

$$P=P_0 \cdot e^{-\mu\alpha-\lambda L} \quad \cdots\cdots(8)$$

P : tensioning force of PC tendon at the beam end which is given by a jack.

P_0 : tensioning force of PC tendon at the section apart L from the beam end

α : change in angle along the length L of PC tendons (radian)

L : length of PC tendons measured from the tensioning end (m)

μ : frictional coefficient per the unit change in angle (=1 radian)

λ : frictional coefficient per the unit length (=1 m)

When the decrease of the tensile stress of PC tendon is small-that is, when the anglular change α and the length L are small, the next approximate equation is sufficient to be used.

$$\frac{P-P_0}{P_0}=-\mu\alpha-\lambda L \quad \cdots\cdots(9)$$

Now, expanding the equation (8) and taking its first term for the practical use, we obtain the equation (10).

When PC tendon is located in the bent as shown in **Fig. 1**, the tensioning force at the inflection point is obtained from the equation (9) and its vertical component is enough to be taken account of. When PC tendon is located in the parabolic as shown in **Fig. 3**, y (mentioned previously) is represented in the following equation.

$$y=ax(x-l) \quad \cdots\cdots(10)$$

where,

a : $4f/l^2$

l : span length of the concrete beam

f : eccentric distance of PC tendon at the center of a beam

Such a case as PC tendon is fixed at the end B and tensioned at the end A, is considered for the first time.

The angular change α along the length from the end A to a point x is expressed as below.

$$\alpha=\left|\left(\frac{dy}{dx}\right)_{x=0}-\left(\frac{dy}{dx}\right)_{x=x}\right| \quad \cdots\cdots(11)$$

Differentiating the equation (10) with respect to x, we have

$$\frac{dy}{dx}=a(2x-l) \quad \cdots\cdots(12)$$

Therefore,

$$\alpha=2ax \quad \cdots\cdots(13)$$

After all, the tensile stress P_x of PC tendon at the point which is apart x from the end A, is represented in the following equation.

$$P_x=\{1-(2a\mu+\lambda)x\}P_0 \quad \cdots\cdots(14)$$

Then the bending moment M_x resulted from prestressing the PC tendon, is given as follows.

$$M_x=P_x \cdot y=aP_0x(x-l)\{1-(2a\mu+\lambda)x\} \cdots(15)$$

The second derivative of M_x with respect to x is obtained as below.

$$\frac{d^2M_x}{dx^2}=2aP_0\{1+(2a\mu+\lambda)l-3(2a\mu+\lambda)x\} \quad \cdots\cdots(16)$$

So the equivalent force to which the concrete beam is subjected, is shown in the equation (17).

$$w_x=-2aP_0\{1+(2a\mu+\lambda)l-3(2a\mu+\lambda)x\} \quad \cdots\cdots(17)$$

Therefore,

w_x at $x=0$ is represented in the following equation,

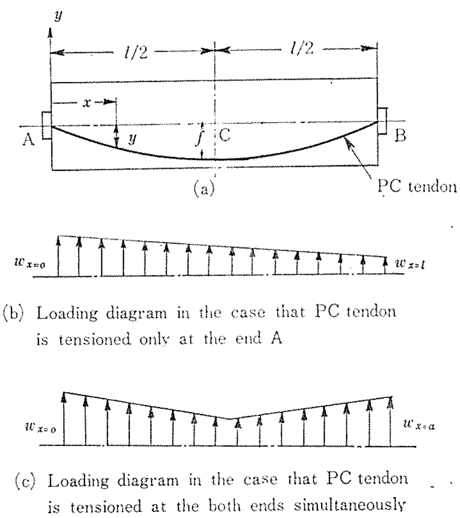
$$w_{x=0}=-2aP_0\{1+(2a\mu+\lambda)l\} \quad \cdots\cdots(18)$$

w_x at $x=l$ is represent likewisely,

$$w_{x=l}=-2aP_0\{1-2(2a\mu+\lambda)l\} \quad \cdots\cdots(19)$$

After all, the equivalent load of a concrete beam

Fig. 3 The equivalent load in the case that a frictional loss is taken account of



(a)

(b) Loading diagram in the case that PC tendon is tensioned only at the end A

(c) Loading diagram in the case that PC tendon is tensioned at the both ends simultaneously is the trapezoidal upward one which is shown in **Fig. 3** (b).

Now, when P C tendon is tensioned in P_0 at the both ends A and B simultaneously, the equivalent load is obtained by substituting $l/2$ in the equations (18) and (19) instead of l. Hence the load can be expressed at each point by the following equations.

$$\left.\begin{array}{l} \text{at the end } A, \\ \quad w_A = 2\,a'P_0\{1+(2\,a'\mu+\lambda\}\,l/2\}\,(\text{upwards}) \\ \text{at the center of the beam,} \\ \quad w_B = 2a'P_0\{1-(2\,a'P_0\{1-(2\,a'\mu+\lambda)l/2\} \\ \qquad\qquad\qquad\qquad\qquad\qquad (\text{upwards}) \\ \text{at the end } B, \\ \quad w_B = 2\,a'P_0\{1+(2\,a'\mu+\lambda\}\,l/2\}\,(\text{upwards}) \end{array}\right\}(20)$$

where, $a'=32f/l^2$

3. The analysis of P C continuous beams

Generally, in statically indeterminate structures the reaction force at a support changes because of having prestressed the P C tendon. Therefore the bending moment and the shearing force acting in that structures should be changed. The difference between a simple and a continuous beam under prestress can be represented by the existence of a secondary moment. Once these moments over the supports are determined, they can be interpolated for any point along the beam. These moments are called secondary ones because they are by-products of prestressing and because they do not exist in a statically determinate beam.

From this point of view, the moment in the concrete given by the eccentricity of the prestress is designated as the primary moment M_0, such as would exist if the beam were simple.

$$M_0 = P \text{ (tensile force of the P C tendon)} \times e \text{ (eccentric distance)} \cdots\cdots(21)$$

On account of such primary moment acting on continuous beam, the secondary moments caused by the induced relations can be computed. The resulting moment due to prestress, is then algebraic sum of the primary and the secondary moments.

Resulting Moment M=Primary Moment M_0 +Secondary Moment M'

In the same way, the resulting shearing forces is given as follows.

$$S = S_0 + S' \cdots\cdots(22)$$

The calculation of the statically indeterminate force produced as a result of prestressing the P C tendon can be performed by the ordinary elastic theory. There are various methods for obtaining this statically indeterminate force.

1) Three moment method, 2) Moment distribution method, 3) The method by the principle of virtual work, 4) The method which uses the relation between the bending moment and the angle of rotation, 5) The method which uses relation between the load and the displacement.

(1) The ordinary slope-deflection equation

Generally, in continuous beams it is very numerous to have haunches at the intermediate supports. In statically indeterminate beams, haunches give such influences to the moment diagram of a beam as they make the negative bending moment increased at the support and the positive bending moment decreased at the center of a span. When the effect of haunches is considerably large, the value of the bending moment of the beam haunched is different from the one of the bending moment which is calculated as the concrete beam with the uniform cross section, and so the effect of haunches must be taken account of. Therefore the slope-deflection equation is expressed as follows. In this case the deflection angle is presumed to be zero.

$$\left.\begin{array}{l} M_{ab} = EK_c\{c_1\theta_a + c_2\theta_b\} + C_{ab} \\ M_{ba} = EK_c\{c_2\theta_a + c_3\theta_b\} + C_{ba} \end{array}\right\}\cdots\cdots(23)$$

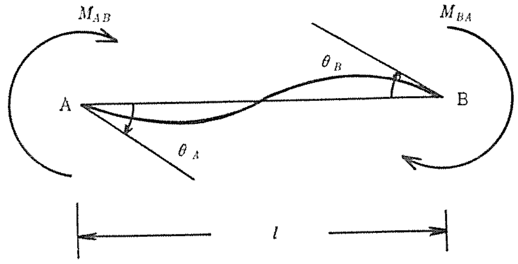
where, c_1, c_2 and c_3 are meant to be coefficients of the beam. Taking the moment of inertia of a haunchless section I_c as standard, I_c/l is meant to be K_c θ_a and θ_b are the slope-deflections of the beam.

4. The experimental analysis by means of analogous circuits

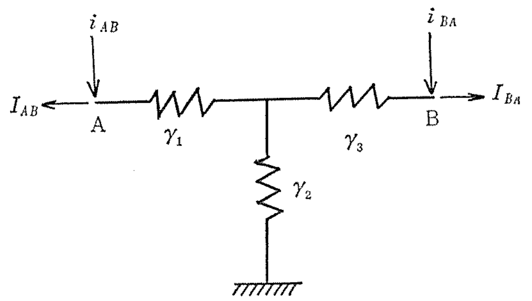
The method for obtaining the bending moment of prestressed concrete beams by means of analogous circuits which consist of electric resistors is described hereinafter.

(1) The similarity between the electrical properties and the elastic ones of P C beams

Fig. 4 Similarity between the electrical properties and the elastic ones of P C beams



(a)



(b)

As showing in **Fig. 4** (b), the electric resistance r_1, r_2 and r_3 are connected together in the shape of T. When electric currents i_{AB} and i_{BA} are applied to each point A and B and the voltages at the same points are measured to be V_A and V_B respectively, the electric currents I_{AB} and I_{BA} run out from each point are represented in the following equations.

$$\left.\begin{aligned} I_{AB} &= \frac{(r_2+r_3)}{(r_1+r_2)\,(r_2+r_3)-r_2^2}(-V_A) \\ &\quad + \frac{r_2}{(r_1+r_2)\,(r_2+r_3)-r_2^2}(V_B)+i_{AB} \\ I_{BA} &= \frac{r_2}{(r_1+r_2)\,(r_2+r_3)-r_2^2}(V_A) \\ &\quad + \frac{r_2}{(r_1+r_2)(r_2+r_3)-r_2^2}(-V_B)+i_{BA} \end{aligned}\right\} \quad (24)$$

Hence, comparing the equation (24) with the equation (23), it is recognized in the following expressions that left sided properties correspond with right sided ones each other;

bending moment M and C
deflection angle θ
stiffness of a beam EK_c
electric current I
electric voltage V
electric resistance r

Furthermore in T shaped circuit, when a positive sign is used for the electric current run in the connected point and a negative sign for the electric one run out from the connected point and the positive or the negative signs are given by turns for all connected points of the circuit. It is obvious that the both equations (23) and (24) coincide with each other, resulted from multiplying a certain constant to I-value and V-value. Therefore, it is practicable to use the circuit shown in **Fig. 4** (b) as the analogous circuit.

(2) The calculation of load terms

In the calculation of the moment of statically indeterminate beams, it is necessary to solve the beam of statically determinate, on the ends of which the moment corresponding to load terms acts. This means only to measure the electric current of the analogous circuit, to the connected points of which the current corresponding to the load terms is applied.

So, it is necessary to obtain the load terms for every kinds of loading on the beam. In some cases of loading, Dr. Takaaki MIZUNO, Dr. R. Guldan and Dr. H. Cross having calculated and reported on the load terms of a beam with haunches, it is not sufficient for this beam to be analyzed by ones mentioned previous. Therefore, for the cases of that the loads shown in **Fig. 6** (a), (b) and (c) act on the beams with haunches as shown in **Fig. 5**, the load terms have been calculated by the authors.

In **Fig. 6** (a), P C tendon is located in the parabolic to the half way from the left end and in the straight on the way left. In **Fig. 6** (b), the curvature of P C tendon is reversed at a certain point. **Fig. 6** (c) shows the case that the frictional loss of P C tendon is considered. The results of calculations for this cases are shown in **Fig. 7, 8, 9** and **10**.

(3) Assemblying the analogous circuit and measuring instruments

Fig. 5 The beam with haunches

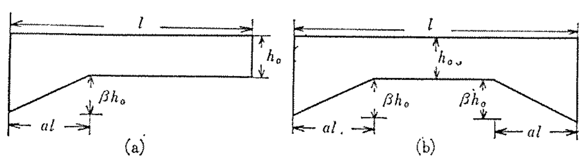


Fig. 6 Some cases of loading

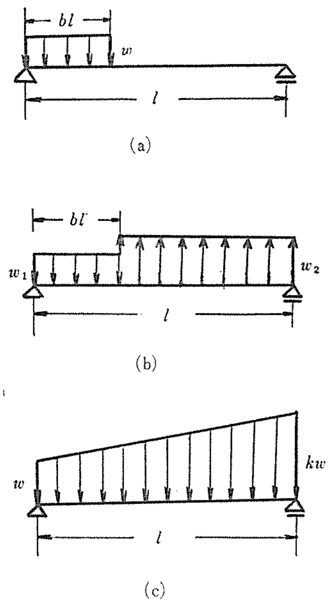
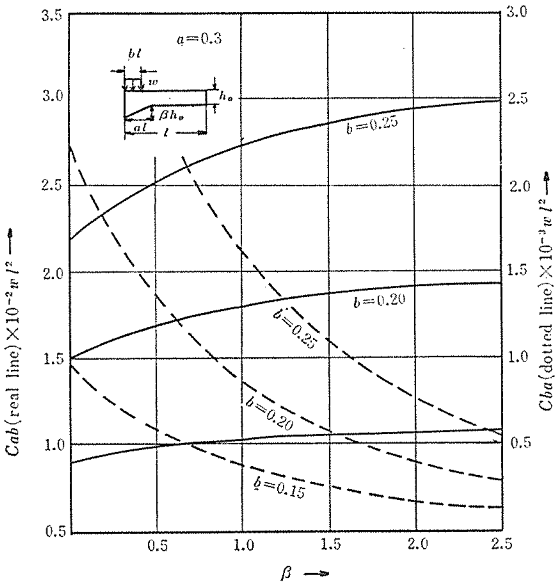


Fig. 7



For the resistors of the analogous circuit, the variable ones of winding type with capacity of 500 Ω are used. To the circuit, direct current is applied in order to keep the accuracy of measuring high degree. For measuring electric voltages, the digital volt-meter is used. By the simple operation of switching, it is performed to put the positive or the negative sign alternately on each connected point and also on the current of the point.

The instrument as shown in the **Photo. 1**, can be utilized for analyzing 1 to 4 span continuous beams, inclusive.

Fig. 8 (a)

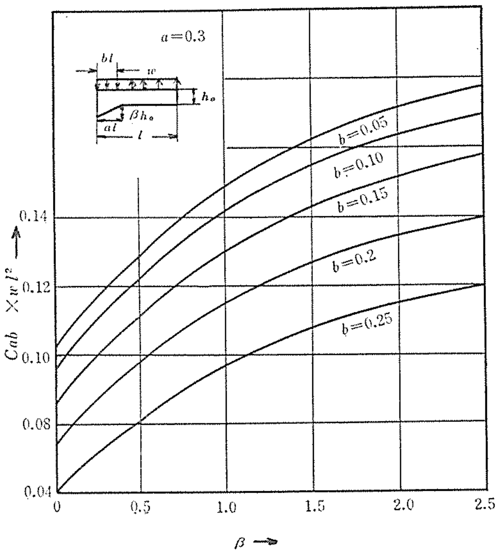


Fig. 8 (b)

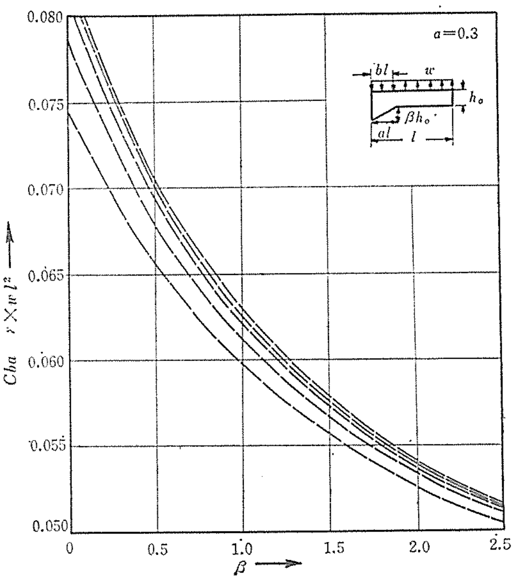


Fig. 9

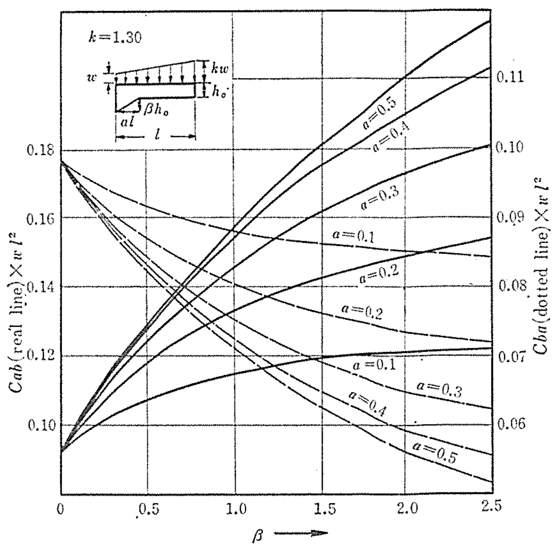


Photo. 1

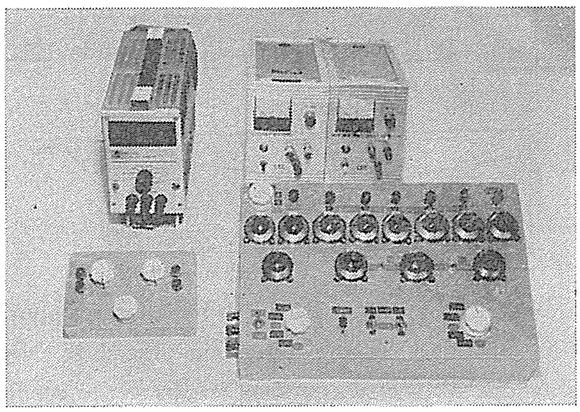
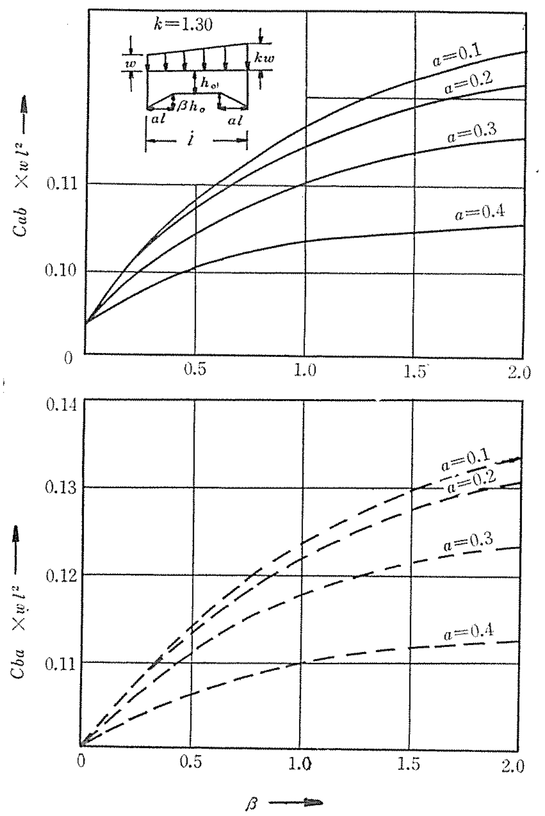


Fig. 10



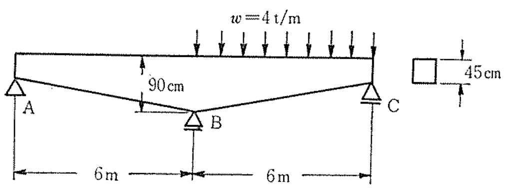
Preventing from that the load acts over 2 spans adjoined each other, it can be attained that the sign of each connected point of circuit coincides with the one of the support of the beam which is subjected to the fixed end moments at its ends.

5. Experimental results

(1) 2 span tapered continuous beam

Let us consider the case that 2 span continuous beam which is shown in **Fig. 11** is of the rectangular cross section with the constant width and the span BC is loaded with the uniform load. This beam is tapered for the both spans and the height of the beam is 45 cm at the cross section A, 90 cm at B section and 45 cm at C section.

Fig. 11 2 span tapered continuous beam.



Now the bending moment at the center support B is obtained by the following procedure. By using the table of R. Guldan, we have the coefficients of the beam, c_1, c_2 and c_3 as below;

for AB span : $c_1=6.8$, $c_2=5.7$, $c_3=19.5$

for BC span : $c_1=19.5$, $c_2=5.7$, $c_3=6.8$

Therefore, for AB span a ratio of ($r_1 : r_2 : r_3$) having to be directly equal to the one of (13.8 : 5.7 : 1.1), the value of ($r_1 : r_2 : r_3$) is directly determined to be (138 Ω : 57 Ω : 11 Ω). This concrete beam being of symmetry, the analogous circuit is of course set to be of symmetry. When the uniform load acts on the beam tapered, the load term C_{bc} and C_{cb} are given as follows by using the table of *H.* Cross and *R.* Guldan.

$$C_{bc}=0.1225\,wl^2,\quad C_{cb}=0.0520\,wl^2$$

Hence, when then the electric current of 0.1225 (A) was applied to the point B and 0.0520 (A) was to the point C, the measured value of valtage was 0.929 (V). So the electric current I_{BA} at the point B is given in the following equation.

$$I_{BA}=\frac{0.929(\mathrm{V})}{11\,(\Omega)}wl^2=0.083\,wl^2$$

Therefore, substituting $w=4$ ton/m and $l=6$ m, the bending moment M_{BA} at the point B is

$M_{BA}=0.083\times144=12.16$ t·m

The theoretical value=11.95 t·m

(2) 2 span PC continuous beam of the constant cross section

The example of a continuous prestressed concrete beam with a bonded tendon as shown in

Fig. 12 2 span PC continuous beam of the constant cross section

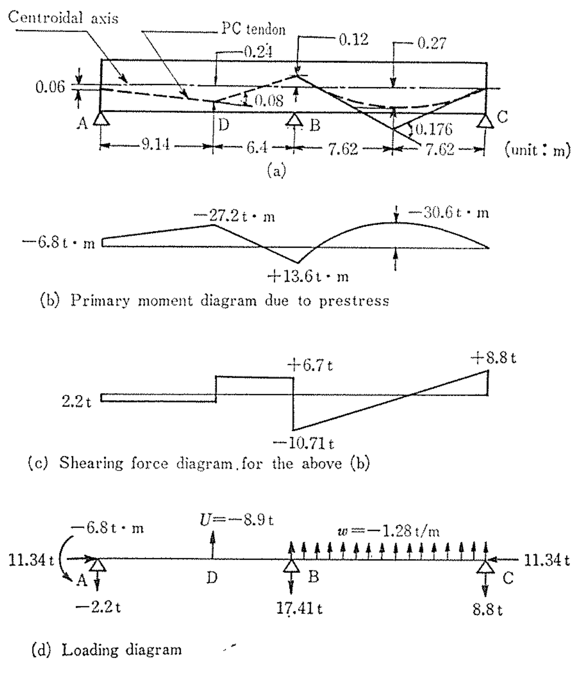


Fig. 12 (a) is treated herein. The c.g.s. has an eccentricity at A, is bent sharply at D and B, and has a parabolic curve for the span BC. In this case, the prestressing force is considered to be 113.4 ton, and the dead load of the beam to be neglected.

For the first time, the primary moment diagram due to prestress is shown in **Fig. 12** (b). Therefore the corresponding shearing force diagram is computed and shown in **Fig. 12** (c), from which the loading diagram is drawn in (d). Owing to this, it is obvious that the concentrating load of 8.9 ton acts upwards at D, that the uniformly distributed load of 1.28 ton/m acts upwards along the span BC and that the anti-clockwise end moment acts at A.

Where, the value 1.28 ton/m is obtained from the equation (7) under the condition of $P=113.4$ ton, $l=15.24$ m, $l_1=7.62$ m, $e=0.12$ m and $f=0.39$ m.

After all, it is sufficient to analyze the continuous beam which is shown in **Fig. 6** (d). The cross section and the stiffness of the beam being constant, the value of the resistance of resistors which are used for the corresponding circuit is all the same.

Now the resistor of 60 Ω which is chosen arbitarily, can well be used. Calculating the load terms due to the concentrating load for the span AB, they are

$$C_{ab}=P\frac{ab^2}{l^2}=0.01464\times10^2 P$$

$$C_{ba}=P\frac{a^2b}{l^2}=0.0219\times10^2 P$$

Hence, DC of 0.01464 (A) is only applied to the point A and 0.0219 (A) is to the point B. Then, the measured value of voltage at B being -0.833 (V), the moment at the same point is computed as below.

$$M_{b_1}=\frac{-0.833(\mathrm{V})}{60(\Omega)}\times10^2 P=-13.09\ (\mathrm{t\cdot m})$$

On the other hand, the load terms due to the uniform load for the span BC are represented as follows.

$$C_{bc}=C_{cb}=\frac{wl^2}{12}=19.4\,w$$

From the result of this, the electric current of 0.194 (A) being enough to be applied to the point B and C and multiplying $10^3 w$ by the measured voltage, the moment at the point B is obtained in the following expression.

$$M_{b2}=\frac{-0.871(\mathrm{V})}{60(\Omega)}\times10^3 w=-18.56\ (\mathrm{t\cdot m})$$

Likewisely, applying DC of -6.8×10^{-2} (A) corresponding to the end moment $M_A=-6.8$ t·m to the point A and multiplying 10^2 by the measured voltage, the moment due to the anti-clockwise end moment at B is obtained;

$$M_{b3}=\frac{-1.02(\mathrm{V})}{60(\Omega)}\times10^2=-1.7\ (\mathrm{t\cdot m})$$

After all, the secondary moment M' due to prestress at B is summarized as below.

$$M'=M_{B_1}+M_{B_2}+M_{B_3}=-33.35\ (\mathrm{t\cdot m})$$

Therefore, the moment M_B acting at B is finally obtained by the following equation.

$$M_B=M_0'+M'=13.60-33.35=-19.75\ (\mathrm{t\cdot m})$$

(3) 2 span P C continuous beam with haunches

Let us consider the case of 2 span P C continuous beam with haunches as shown in **Fig. 13** (a).

The moment at the support B can be obtained by the following procedure, under the conditions of that the prestressing force is $P=50$ ton, that the frictional loss of P C tendon is presumed to be negligible and that the dead weight of a concrete beam not to be considered.

1) The primary moment due to prestress (cf. **Fig. 13** (b))

The primary moments due to prestress are represented as follows.

at the point A $0.05\times50=2.5$ t·m

at the center of span AB

$-0.1\times50=-5.0$ t·m

at the point B $9.75\times10^{-2}\times50=4.9$ t·m

at the point C $9.75\times10^{-2}\times50=4.9$ t·m

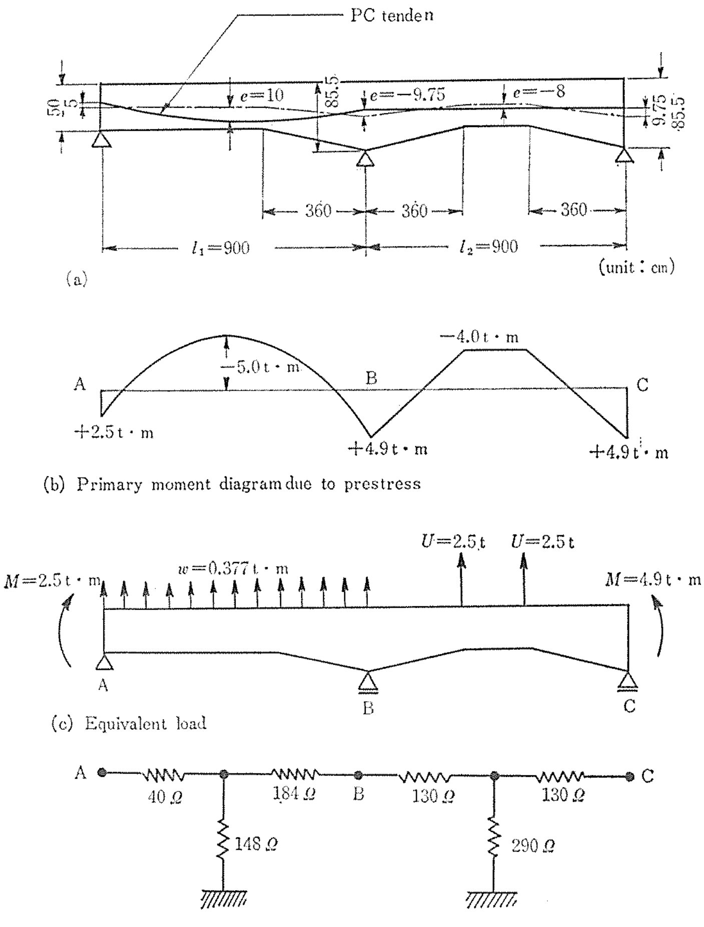
2) The loading diagram for the moment diagram in (b)

(a) AB span : From the equation (7) under the conditions of $P=50$ ton, $l=9$ m, $e=0.15$ m and $l_1=4.5$ m, the uniform load w is given by

$w=0.377$ t/m (upwards)

(b) BC span : Each concentrating load of the same value is

Fig. 13 2 span PC continuous beam with haunches.



(d) Analogous circuit

$$U=50\times\frac{(9.75+8.00)}{360}=2.5\ \text{t (upward)}$$

(c) A point : At the point A, PC tendon being located by the eccentric distance of 0.05 m, the end moment M is

$M=2.5$ t·m

(d) C point : PC tendon also produces the eccentric moment at C as well as at A, that is

$M=4.9$ t·m

From the results described above, it becomes clear that the beam, shown in **Fig. 13** (d), is enough to be solved.

3) The calculation of the coefficient of the beam.

The results of this calculation are listed in the **Table 1**.

4) The determination of the resistance value of resistors for the analogous circuit.

The ratio of each resistance must satisfy the following equations respectively.

for the span AB,

Table 1 Coefficients of the beam

Span	Coefficients		
	C_1	C_2	C_3
AB	8.3	3.7	4.7
BC	10.8	7.2	10.8

$$c_1 : c_2 : c_3=(r_2+r_3) : r_2 : (r_1+r_2)$$
$$=40\,\Omega : 148\,\Omega : 184\,\Omega$$

for the span BC, likewisely,

$$c_1 : c_2 : c_3=130\,\Omega : 290\,\Omega : 130\,\Omega$$

Therefore the analogous circuit is finally obtained as shown in **Fig. 7** (d).

5) The calculation of load terms

The load terms can be attained by means of the table and the graph of R. Guldan's and H. Cross's. The results of this are listed in the **Table 2**.

6) The experimental results

The measured values of the secondary moment M' at the support B is listed in the **Table 3**. According to the **Table 3**, the moment at the support B is computed to be

$$M_B=M_0+M_B'=4.90+6.43=11.33 \quad (\text{t·m})$$

Table 2 Load terms

Span	Load	Load terms (t·m)
AB	Uniform load $w=0.377$ t/m	$C_{ab}=0.788\times\frac{wl^2}{12}=2.000$ $C_{ba}=1.496\times\frac{wl^2}{12}=3.803$
BC	Concentrating load $U=2.5$ t	$C_{bc}=0.380\ Ul=8.430$ $C_{cb}=0.240\ Ul=5.324$

Table 3 Measured results

Load	Currents (A)	Measured values (t·m)
Uniform load	at A 0.020 B 0.038	$\frac{+2.68(\text{V})}{130\ (\Omega)}\times10^2=+2.06$
Concentrating load	at B 0.00843 C 0.00532	$\frac{+1.241(\text{V})}{184\ (\Omega)}\times10^3=+6.74$
Load due to premoment at A	at A 0.025	$\frac{-0.638(\text{V})}{130\ (\Omega)}\times10^2=-0.49$
Load due to premoment at B	at C 0.0488	$\frac{-3.46(\text{V})}{184\ (\Omega)}\times10^2=-1.88$
		$M_B'=+6.43$ t·m

7) Considerations

Comparing this value with the ones calculated by other methods, we have

		(Moment)
1	Moment Distribution Method	11.49 t•m
2	Three Moment Method	11.64 t•m

The value measured by this method is accurate to within about 3% of the values calculated theoretically.

6. Conclusion

The value of moments measured by this method is accurate to within about 3% of the values calculated theoretically.

As the results of this experiment, it is provided that a prestressed concrete beam can be analyzed easily with high accuracy by replacing the internal force of the beam by the external one corrresponding to the former and besides by using this analogous circuit.

The authors carried out the experiment only for 2 span P C continuous beam. However, it is believed to attain the more superiority of using this method, with increase of the number of span.

Hereafter, there will be the necessities for calculating the load terms due to any loading on the beam with any section. And the analogous circuit corresponding to any P C structure having to be studied, it will be very convenient to design P C structures.

The authors are thankful to Research Assistant Shoji SASAKI of Kyushu Institute of Technology for his assistance in carrying out electrical measurements.

The authors who have much to learn yet would deeply appreciate if this work should be given criticism widely by the readers.

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1970.3.20・受付

DESIGN AND EXECUTION OF NEW CONSTRUCTION WORK OF MAOKA PLANT OF IKEDA BUSSAN CO. LTD.

By T. WATANABE, Y. OKITA AND Y. UCHIDA (Page 54)

This plant was planned to construct in some part of Maoka Industrial Group of Japan Housing Corporation in Maoka City of Tochigi Pref. Its constructed system is, after several investigation, first floor is used together with reinforced concrete and fabric prestressed concrete structures and second floor is steel frame structures were adopted. This report describe principally about its design and execution of prestressed concrete parts.

PRESTRESSED CONCRETE CONSTRUCTION OF TOCHIGI PREFECTURAL ASSEMBLY HOUSE

By M. YAMAGA, Y. MORIMURA AND T. TATAI (Page 60)

This is a continued report of Vol. 11, No. 5, and this time we describe about construction of 2 500 tons prestressed concrete agents including large type agent of 1 200 tons.

CONDITIONS OF USING ELECTRONIC COMPUTER FOR DESIGNING IN P C MAKER AND TROUBLES OF DERIVING FROM USING

By S. Hosaka (Page 66)

This report treats of following questions. In P C maker, why must be used electronic computer for designing P C bridges How ?. How to use and avoid from troubles of deriving from using ?.